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# BRIEF COMMUNICATION

# CLUSTERING UNDER THE INFLUENCE OF ELECTROSTATIC FORCES

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**Abstract**—Using the approach of Jackson and Grace & Tuot to analyze the ability of a gas–solid transport to form clusters of particles, an electrostatic force was incorporated into the analysis. The analysis does not include any frictional forces due to particle–particle and particle–wall interactions. Electrostatic forces are shown to decrease the distance in which clusters will form. The effect is strongly influenced by the particle size, with smaller particles having shorter growth distances. The effect of the parameters gas velocity, voidage and charge are explored.

## INTRODUCTION

Analysis of the stability of the fluidization and gas–solid transport process led first Jackson (1963) and then Grace & Tuot (1979) to investigate the basic transport equations, in terms of the number density, when written in a linearized form. The basic continuity equations for the particles and fluid are

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} nv = 0 \qquad [1]$$

and

$$v_p \frac{\partial n}{\partial t} - \frac{\partial}{\partial x}(1 - nv_p)u = 0, \qquad [2]$$

where $n$ is the number density of the particles, $u$ is the fluid velocity, $v$ is the solid velocity and $v_p$ the particle volume. The number density is defined in terms of the voidage, $\epsilon$,

$$n = \frac{(1-\epsilon)}{v_p}. \qquad [3]$$

Considering vertical upward flow the dynamic equation for the solids can be written neglecting particle–particle and particle–wall friction, as

$$n\, m_p\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right) = -nv_p\frac{\partial P}{\partial x} - n\,(m_p - v_p\rho_f)g - F_e + \beta(n)\,(u - v). \qquad [4]$$

The term involving ($\beta$) is the drag force and $F_e$ is the electrostatic force on the system.

For the fluid, neglecting friction, the momentum equation is

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{-1}{\rho_f}\frac{\partial P}{\partial x}. \qquad [5]$$

Inserting the pressure drop term from [5] into [4]:

$$nm_p\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right) - \rho_f nv_p\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) + n(m_p - v_p\rho_f)g + F_e - \beta(n)\,(u - v) = 0. \qquad [6]$$

A perturbation analysis is now applied to [6] using only the linear terms. For the perturbation analysis,

$$u = u_0 + u_1,$$

$$v = v_0 + v_1$$

and

$$n = n_0 + n_1.$$

One obtains on substitution and simplification [1], [2] and [6]:

$$\frac{\partial v_1}{\partial x} = -\frac{1}{n_0}\left(\frac{\partial n_1}{\partial t} + v_0 \frac{\partial n_1}{\partial x}\right), \quad [7]$$

$$\frac{\partial u_1}{\partial x} = \frac{v_p}{1 - n_0 v_p}\left(\frac{\partial n_1}{\partial t} + u_0 \frac{\partial n_1}{\partial x}\right), \quad [8]$$

$$n_0 m_p\left(\frac{\partial v_1}{\partial t} + v_0 \frac{\partial v_1}{\partial x}\right) - n_0 \rho_f v_p\left(\frac{\partial u_1}{\partial_t} + u_0 \frac{\partial u_1}{\partial x}\right) + g n_1 (m_p - \rho_f v_p) \quad [9]$$

and

$$- \beta(n_0)(u_1 - v_1) + F_e - n_1 \beta'(n_0)(\mu_0 - v_0) = 0,$$

where

$$\beta'(n_0) = \frac{d\beta}{dn}.$$

Jackson (1963) combined [7], [8] and [9] by taking the divergence of [9], using [7] and [8] accordingly, and subtracting the steady-state equation. Using the relations

$$m_0 = \rho_p v_p,$$

$$\epsilon_0 = 1 - n_0 v_p,$$

$$\beta(n_0) = \frac{(1-\epsilon_0)(\rho_p - \rho_f) g}{u_0 - v_0}$$

and

$$F_e = n_1 Q,$$

where the term $Q$, accounting for the electrostatic contribution, was shown by Ally (1980) to be

$$Q = \frac{\lambda^2 D_t n_0}{\zeta} \quad [10]$$

where

$$\lambda = \text{charge/particle},$$

$$D_t = \text{diameter of the tube}$$

and

$$\zeta = \text{permittivity of air},$$

one obtains

$$\left[\left(\frac{\epsilon_0}{1-\epsilon_0}\right)\frac{\rho_p}{\rho_f} + 1\right]\frac{\partial^2 n_1}{\partial t^2} + \left[v_0^2 \frac{\rho_p}{\rho_f}\left(\frac{\epsilon_0}{1-\epsilon_0}\right) + u_0^2\right]\frac{\partial^2 n_1}{\partial x^2}$$
$$+ \left[\frac{u_0 - 2\epsilon_0(u_0 - v_0)}{\epsilon_0(u_0 - v_0)} + \frac{\beta'(n_0)^1}{\beta(n_0)} n_0\right]\frac{\partial n_1}{\partial x} + \frac{\epsilon_0}{(1-\epsilon_0)\rho_f v_p} Q \frac{\partial n_1}{\partial x}$$
$$+ \frac{(\rho_p - \rho_f) g}{(u_0 - v_0)\rho_f(1-\epsilon_0)}\frac{\partial n}{\partial t} + 2\left[\frac{\rho_p v_0 \epsilon_0}{\rho_f(1-\epsilon_0)} + 2u_0\right]\frac{\partial n_1^2}{\partial x \partial t} = 0. \quad [11]$$

The various factors can be combined in convenient form as

$$a = \frac{1 + \rho_p \epsilon_0}{\rho_f(1-\epsilon_0)},$$

$$b = \frac{(\rho_p - \rho_f) g}{\rho_f(1-\epsilon_0)(u_0 - v_0)}$$

and

$$c = k(u_0 - v_0),$$

with $k$ as the growth constant

$$e = \frac{1 - 2\epsilon_0 + \epsilon_0 n_0 \beta'(n_0)}{\beta(n_0)},$$

$$q = \frac{v_0 a}{(u_0 - v_0)}$$

and

$$d = \frac{\epsilon_0}{(1 - \epsilon_0)\rho_f v_p} Q.$$

Combining the above terms, [11] can be written as

$$\alpha \frac{\partial^2 n_1}{\partial t^2} + \frac{c}{k^2}\left(1 + 2\frac{q}{a} + \frac{q^2}{a}\right)\frac{\partial^2 n_1}{\partial x^2} + \left[\frac{bc}{k}\left(\frac{q}{a} + e\right) + Q\right]\frac{\partial n_1}{\partial x_1} + \frac{b \partial n_1}{\partial t} + \frac{2c}{k}(1 + q)\frac{\partial^2 n_1}{\partial x \partial t} = 0. \quad [12]$$

Using the classic perturbation analysis format of

$$n_1 = f(t)\exp(jkx), \quad [13]$$

this can be inserted into [10]. After much algebra and solving the second-order linear differential in $f(t)$, one obtains solution in exponential form as

$$\exp(s_1 t) \quad \text{and} \quad \exp(s_2 t).$$

The roots $s_1$ and $s_2$ are complex and are given as

$$s_1, s_2 = \frac{b}{2a}\left\{\pm\sqrt{1 + 4(a-1)\frac{c^2}{b^2} - 4j\frac{c}{b}(ae - 1) - 4j\frac{ak}{b^2}d} - \left[1 + 2j\frac{c}{b}(1 + q)\right]\right\}. \quad [14]$$

In order to analyze the cluster behavior Jackson expressed the growth distance in the vertical distance as

$$\Delta x = -\frac{\mathcal{I}m(s_1)}{k\,\mathcal{R}e(s_1)}. \quad [15]$$

To evaluate growth distances several other aspects must be considered. The growth constant ($k$) is assumed proportional to the particle diameter. The constant is defined as

$$k = \frac{2\pi}{l} = \frac{\pi}{Nd_p}. \quad [16]$$

Grace & Tuot (1979) have taken $N = 50$, where $N$ is the number of particles in the disturbance. For the drag term the factor $\beta(n_0)$ is taken from the Richardson & Zaki (1954) equation:

$$\beta(n_0) = \frac{(\rho_p - \rho_f)g(1 - \epsilon_0)}{U_T \epsilon_0^{\eta - 1}}, \quad [17]$$

where

$$U_T = \text{terminal velocity}$$

and

$$\eta = 4.7.$$

Other drag formats could also be employed. The derivative of this equation must also be evaluated. One finds, applying the above,

$$\frac{n_0 \beta'(n_0)}{\beta(n_0)} = \frac{(\eta - 1) - \epsilon_0(\eta - 2)}{\epsilon_0}. \quad [18]$$

The voidage ($\epsilon_0$) is that at minimum fluidization and is assumed to be 0.43.

Table 1. System parameters

| |
|---|
| $u_0 = 5,\ 10,\ 25\ \mathrm{m/s}$ |
| $\epsilon = 0.99,\ 0.95,\ 0.90,\ 0.80$ |
| $d_p = 50,\ 100,\ 250\ \mu\mathrm{m}$ |
| $Q = 0,\ 10^{-12},\ 10^{-14}$ C/particle |
| $\rho_p = 2340\ \mathrm{kg/m^3}$, $\rho_f = 1.2\ \mathrm{kg/m^3}$ |
| $D_t = 0.0254\ \mathrm{m}$ |

Using the above, growth distances can be investigated for a number of different cases. It should be noted that [10] shows that the electrostatic contribution is proportional to $n_0$, which in turn is inversely related to the cube of the particle diameter. The particle diameter thus has a strong influence on the electrostatic force contribution.

For analysis, the parameters associated with gas–solid transport and the electrostatic forces encountered were varied. The values of the charges are seen from $10^{-12}$ to $10^{-14}$C. This range is consistent with the data in our laboratory and other laboratories. The overall objective was to examine how these parameters influence the growth distances, and hence cluster formation, in a gas–solid system. Table 1 shows the range of the parameters explored. Rather than specifying the solid's flow rate, the system's voidage is stated.

## RESULTS AND DISCUSSION

Figures 1–3 show the findings of the parametric study. The growth distance covers several orders of magnitude, thus the findings are reported in a semi-logarithmic format. Figure 1 was prepared for a superficial gas velocity of 5 m/s. For zero charging effect, as the particle size increases the growth distance decreases, and for a given particle size it decreases as the voidage decreases. The more particles present and the larger the particles, the less the growth distance. With a charge of $10^{-14}$ C/particle a unique behavior was observed at the low gas velocity of 5 m/s. Overall the effect of charging is to decrease the growth distance. The effect of particle size, however, is most pronounced. As mentioned previously, the larger the particle size, the smaller the electrostatic force. For the charge of $10^{-14}$ C/particle, the growth distance increased with particle size up to about 100 $\mu$m than started to decrease—showing that electrostatic forces decreased and the particle size effect begins to dominate. For the charge of $10^{-12}$ C/particles, over the range studied the electrostatic force dominated but a decreasing slope growth distance with particle size showed decreasing electrostatic force as the particle size increased.

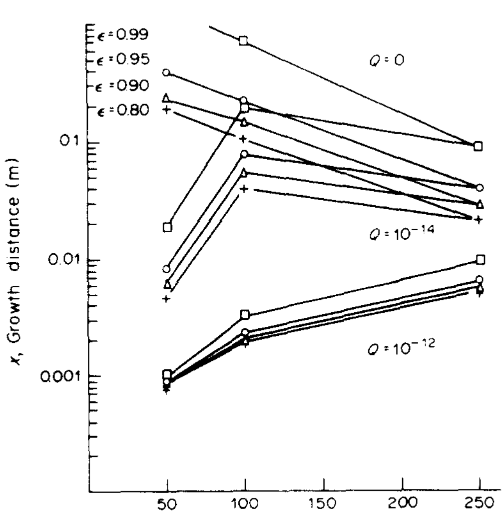


Figure 1. Growth distance vs particle diameter for varying charging, $u = 5$ m/s. □, $\epsilon = 0.99$; ○, $\epsilon = 0.95$; △, $\epsilon = 0.90$; +, $\epsilon = 0.80$.

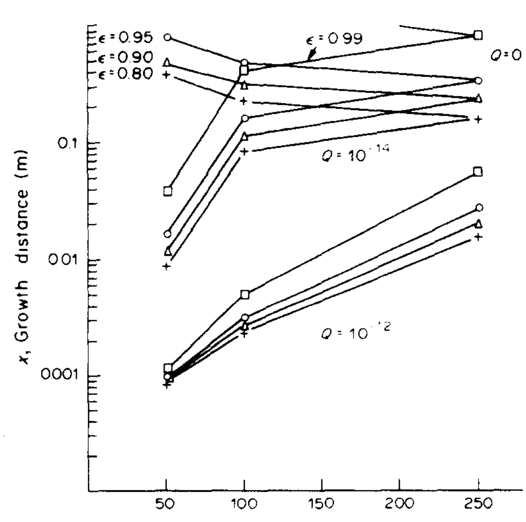


Figure 2. Growth distance vs particle diameter for varying charging, $u = 10$ m/s. □, $\epsilon = 0.99$; ○, $\epsilon = 0.95$; △, $\epsilon = 0.90$; +, $\epsilon = 0.80$.

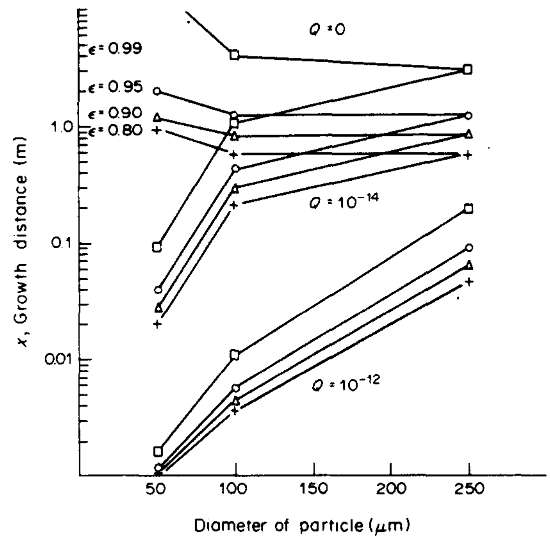


Figure 3. Growth distance vs particle diameter for varying charging, $u = 25\,\mathrm{m/s}$. □, $\epsilon = 0.99$; ○, $\epsilon = 0.95$; △, $\epsilon = 0.90$; +, $\epsilon = 0.80$.

For the gas velocities of 10 and 25 m/s, figures 2 and 3, respectively, no maximum in growth length, due to the electrostatic and gravity balancing was observed. The inertial energy of the particles at these higher gas velocities kept the growth distance in the increasing mode with particle size. For charging of $10^{-14}$ C/particle at approx. 200 μm particle size the electrostatic force becomes negligible compared with the other dynamic forces acting on the particle.

One thus sees that the dynamic state of the flow system is influenced by electrostatic and gravity forces controlling the stability of such flows.

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## REFERENCES

ALLY, M. R. 1980 Electrostatic effects in dilute and dense phase gas–solid pneumatic transport. Ph.D. Dissertation, Univ. of Pittsburgh, Pittsburgh, Pa.

GRACE, J. R. & TUOT, J. 1979 A theory for cluster formation on vertically conveyed suspensions of intermediate density. *Trans Inst. chem. Engrs* **57**, 49–54.

JACKSON, R. 1963 The mechanics of fluidised beds. Part 1: the stability of the state of uniform fluidisation. *Trans. Inst. chem. Engrs* **41**, 13–21.

RICHARDSON, J. F. & ZAKI, W. N. 1954 Sedimentation and fluidization. Part I. *Trans. Inst. chem. Engrs* **32**, 35–53.